Problems and results for the ninth week

Mathematics A3 for Civil Engineering students

1. Production line I of a factory works 60% of time, while production line II works 70% of time, independently of each other. What is the probability that
   (a) both lines operate,
   (b) at least one of the lines operates,
   (c) precisely one of the lines operates,
   (d) both lines are stopped?
2. Suppose that each child born to a couple is equally likely to be a boy or a girl independent of the sex distribution of the other children of the family. For a couple having 5 children, compute the probabilities of the following events:
   (a) All children are of the same sex.
   (b) The 3 eldest are boys and the others girls.
   (c) Exactly 3 are boys.
   (d) The 2 oldest are girls.
   (e) There is at least 1 girl.
3. We roll a red and a green die. Consider the following events: $A = \{$the sum of the numbers shown is 7$\}$, $B = \{$at least one of the dice shows a 6$\}$, $C = \{$both dice show odd numbers$\}$, $D = \{$the dice show different numbers$\}$, $E = \{$the green die shows a 4$\}$.
   (a) Are the events A and C independent?
   (b) Are the events A and C mutually exclusive?
   (c) What is the probability of B?
   (d) How are A and D related? What consequences follow for their probabilities? And for their independence?
   (e) Are the events A and E independent?
   (f) Based on the above, show examples of
      i. independent but not mutually exclusive events,
      ii. mutually exclusive but not independent events.
4. We roll a red and a green die. Consider the following events: $A = \{$the sum of the numbers shown is 7$\}$, $B = \{$the red die shows a 3$\}$, $C = \{$the green die shows a 4$\}$.
   (a) Are the events A and B independent? The events A and C? How about B and C?
   (b) Are the events A, B, C independent?
   (c) Are the events A and $B \cap C$ independent?
   (d) So, are A, B, C independent?

5*. Suppose that we want to generate the outcome of the flip of a fair coin but that all we have at our disposal is a biased coin which lands on heads with some unknown probability p that need not to be equal to 1/2. Consider the following procedure for accomplishing our task.

(1) Flip the coin.

(2) Flip the coin again.

(3) If both flips land heads or both land tails then return to step (1).

(4) Otherwise, if the two flips are different, then let the result of the last flip be the result of the experiment.

(a) Show that the result is equally likely to be either heads or tails.

(b) Could we use a simpler procedure that continues to flip the coin until the last two flips are different and then lets the result be the outcome of the final flip?

6. Independent flips of a coin that lands on heads with probability p are made. What is the probability that the first outcomes are

   (a) H, H, H, H,

   (b) T, H, H, H?

   (c*) What is the probability that the pattern (T, H, H, H) occurs before the pattern (H, H, H, H)? (Hint: How can the pattern (H, H, H, H) occur first?)

7. A true-false question is to be posed to a husband and wife team on a quiz show. Both the husband and the wife will, independently, give the correct answer with probability p. Which of the following is a better strategy for this couple?

   (a) Choose one of them and let that person answer the question; or

   (b) have them both consider the question and then either give the common answer if they agree or, if they disagree, flip a coin to determine which answer to give?

8. Between cities A, B, C, there are the following roads: $A-B$, $A-C$ and $B-C$. At a winter night, each of these road gets blocked by the snow independently with probability p. What is then the probability that on the next morning city C will be accessible from city A?

9. Between cities A, B, C, there are two independent roads between cities A and B, and another road between B and C. If each of these roads gets blocked independently with probability q, then what is the probability that city C will be accessible from city A?

10. Five men and 5 women are ranked according to their scores on an examination. Assume that no two scores are alike and all 10! possible rankings are equally likely. Let X denote the highest ranking achieved by a woman (for instance, X=1 if the top-ranked person is female). Find the probability mass function $\mathbb{P}\{X = i\}$, $i = 1, 2, \ldots, 10$ of X.

11. The mass function of X is given by $p(i) = \frac{i^2}{30}$, $i = 1, 2, 3, 4$. Compute the expectation of X.

12. Adam and Bob play the following game: they both roll a die, and Bob gives Adam an amount equal to the square of the difference of the numbers shown on the two dice. Adam gives Bob an amount equal to the sum of the two numbers shown on the dice. Who is favored by this game?

13. On a raffle they have one HUF1 000 000, 10 HUF50 000, and 100 HUF5 000 prizes. They produce 40 000 tickets for this raffle. What should the price of one ticket be, so that the expected return agrees to the half of that price?

14. Assume the fixed prizes of HUF700, HUF10 000, HUF789 thousand, and HUF535 million on the lottery (5 winner numbers out of 90). With a price HUF150 of a lottery ticket, what are our expected winnings on one lottery ticket?

15. Ann and Bob play with two dice. Ann pays Bob whenever both dice show odd numbers. Bob pays Ann whenever exactly one die shows an even number. In any other case, no payments are made. What amounts should they set up to make this game fair?

16. Let X be the outcome of rolling a die. Compute the expectation and standard deviation of X. What if the 'die' has n sides?

17. A total of 4 buses carrying 148 students from the same school arrive at a football stadium. The buses carry, respectively, 40, 33, 25, and 50 students. One of the students is randomly selected. Let X denote the number of students that were on the bus carrying this randomly selected student. One of the 4 bus drivers is also randomly selected. Let Y denote the number of students in her bus.

    (a) Which of $\mathbb{E}(X)$ or $\mathbb{E}(Y)$ do you think is larger? Why?

    (b) Compute $\mathbb{E}(X)$ and $\mathbb{E}(Y)$.

    (c) Find $\mathbf{Var}(X)$ and $\mathbf{Var}(Y)$.

18. We draw without replacement 3 balls from an urn that contains 4 red and 6 white balls. Denote by X the number of red balls drawn. Find the distribution of X, its expectation and standard deviation.

19. We randomly place a knight on an empty chessboard. What is the expected number of his possible moves? (A knight placed on the square (i, j) of the chessboard can move to any of the squares $(i+2, j+1)$, $(i+1, j+2)$, $(i-1, j+2)$, $(i-2, j+1)$, $(i-2, j-1)$, $(i-1, j-2)$, $(i+1, j-2)$, $(i+2, j-1)$, provided they are on the chessboard.)

20. Rolling two dice, what is the expected value of the higher and of the smaller of the two numbers shown?

21. One of the numbers 1 through 10 is randomly chosen. You are to try to guess the number chosen by asking questions with „yes-no" answers. Compute the expected number of questions you will need to ask in each of the two cases:

    (a) Your ith question is to be „Is it i?", $i = 1, 2, \ldots, 10$.

    (b) With each question you try to eliminate one-half of the remaining numbers, as nearly as possible. For example, your first question is „Is the number larger than 5?". If yes, then your second question is „Is the number greater than 7?", etc.

22. If $\mathbb{E}(X) = 1$ and $\mathbf{Var}(X) = 5$, find

    (a) $\mathbb{E}[(2 + X)^2]$,

    (b) $\mathbf{Var}(4 + 3X)$.

23. Let X be a random variable having expected value μ and variance σ^2. Find the expected value and variance of

$$Y := \frac{X - \mu}{\sigma}.$$

24. How many people are needed so that the probability that at least one of them has the same birthday as you is greater than 1/2?

Answers

1.(a) $0.6 \cdot 0.7 = 0.42$

(b) $1 - 0.4 \cdot 0.3 = 0.88$

(c) $0.6 \cdot 0.3 + 0.4 \cdot 0.7 = 0.46$

(d) $0.4 \cdot 0.3 = 0.12$

2.(a) $(1/2)^5 + (1/2)^5 = 1/16$

(b) $1/32$

(c) $\binom{5}{3} \cdot \left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^2 = 5/16$

(d) $\frac{1}{2} \cdot \frac{1}{2} = 1/4$

(e) $1 - \left(\frac{1}{2}\right)^5 = 31/32$

3.(a) No, because

(b) they are mutually exclusive events.

(c) $1 - \mathbb{P}\{\text{none of them is } 6\} = 1 - \left(\frac{5}{6}\right)^2$.

(d) A implies D, that is $A \subset D$. Hence $\mathbb{P}\{A\} \le \mathbb{P}\{D\}$, and e.g. $5/6 = \mathbb{P}\{D\} \neq \mathbb{P}\{D \mid A\} = 1$ shows that A and D cannot be independent.

(e) The event A is realized in 6 cases out of the 36 possible outcomes, hence its probability is $1/6$. The event E also has probability $1/6$, and $A \cap E$ is realized in precisely one case, its probability is $1/36$. Therefore, $\mathbb{P}\{A\} \cdot \mathbb{P}\{E\} = \mathbb{P}\{A \cap E\}$, the two events are independent.

(f) i. A and E;

ii. A and C.

4.(a) $\mathbb{P}\{A\} = \mathbb{P}\{B\} = \mathbb{P}\{C\} = 1/6$, and $\mathbb{P}\{AB\} = \mathbb{P}\{AC\} = \mathbb{P}\{BC\} = 1/36$, hence A and B; A and C; B and C are pairwise independent.

(c) Since $B \cap C \subset A$, $B \cap C$ and A cannot be independent.

(b),(d) A, B, C are not independent. Our result in (c) is an indication of this fact, and also $1/36 = \mathbb{P}\{A \cap B \cap C\} \neq \mathbb{P}\{A\} \cdot \mathbb{P}\{B\} \cdot \mathbb{P}\{C\} = 1/216$.

5*.(a) The procedure can be reformulated in the following way: We flip the coin twice. If the outcomes agree, then our flips do not matter, we restart the experiment. If the two outcomes disagree, then we consider the second outcome. From this formulation one can see that the outcomes of our procedure follow a conditional distribution:

$$\begin{aligned}\mathbb{P}\{\text{the procedure gives } H\} &= \mathbb{P}\{(T,\, H) \mid (T,\, H) \text{ or } (H,\, T)\} \\ &= \frac{\mathbb{P}\{(T,\, H)\}}{\mathbb{P}\{(T,\, H) \text{ or } (H,\, T)\}} = \frac{(1-p)p}{(1-p)p + p(1-p)} = \frac{1}{2}.\end{aligned}$$

Similarly, the procedure gives tails with probability $1/2$.

(b) For any $0 < p < 1$, we will surely find both heads and tails eventually. Thus method (b) gives H if and only if the first flip comes out T, which happens with probability $1-p$. The method gives T if and only if the first flip is H, this happens with probability p. Hence method (b) does not give fair coin flip results.

The difficulty in this problem is figuring out why our arguments in (a) do not work in the case of method (b). In the above formulation of (a) we consider a *fixed* pair of coin flips (the first two flips), while in method (b) we take a *randomly selected* pair of coin flips, those where we first see a change in the outcomes. This innocent-looking difference essentially modifies the probabilities of the outcomes $(H,\,T)$ and $(T,\,H)$. Our arguments in (a), applied on case (b), would look like this:

$$\begin{aligned}\mathbb{P}\{\text{the procedure gives } H\} &= \mathbb{P}\{(T,\,H)\,|\,(T,\,H) \text{ or } (H,\,T)\}\\ &= \frac{\mathbb{P}\{(T,\,H)\}}{\mathbb{P}\{(T,\,H) \text{ or } (H,\,T)\}} = \frac{(1-p)}{(1-p)+p} = 1-p,\end{aligned}$$

and similarly the probability of outcome T is p.

6.(a) p^4

(b) $(1-p)\cdot p^3$

(c*) Our first observation is that for any $0 < p < 1$ we will eventually see the pattern $(H,\,H,\,H,\,H)$. Suppose that the first such pattern starts at the nth flip. If $n > 1$, then the $n-1$th flip cannot be H since then the first $(H,\,H,\,H,\,H)$ pattern would have started before the nth flip. Hence in this case the $n-1$th flip is necessarily T, and starting with the $n-1$th flip we see the pattern $(T,\,H,\,H,\,H,\,H)$. In that sequence, $(T,\,H,\,H,\,H)$ appears before $(H,\,H,\,H,\,H)$. Summarizing, $(H,\,H,\,H,\,H)$ can only appear before $(T,\,H,\,H,\,H)$ if it starts immediately at the $n = 1$st flip, that is if all four first flips are heads. The probability of that is p^4.

7. In case (a) the correct answer is given with probability p. In case (b), define E as the event that the team gives the correct answer, C and W, respectively, the events that the wife or the husband would give the correct or the wrong answer (that is, e.g. $(C,\,C)$ is the event that they both would give the correct answer). Then

$$\begin{aligned}\mathbb{P}\{E\} &= \mathbb{P}\{E\,|\,(C,\,C)\}\cdot\mathbb{P}\{(C,\,C)\} + \mathbb{P}\{E\,|\,(C,\,W)\}\cdot\mathbb{P}\{(C,\,W)\}\\ &\quad + \mathbb{P}\{E\,|\,(W,\,C)\}\cdot\mathbb{P}\{(W,\,C)\} + \mathbb{P}\{E\,|\,(W,\,W)\}\cdot\mathbb{P}\{(W,\,W)\}\\ &= 1\cdot p^2 + \frac{1}{2}\cdot p(1-p) + \frac{1}{2}\cdot(1-p)p + 0\cdot(1-p)^2 = p.\end{aligned}$$

There is no difference between the two strategies. A strategy similar to (b) would be favorable for larger teams (and $p > 1/2$).

8. Denote the event in question by $A \leftrightsquigarrow C$, and the event that the road $A-B$ is not blocked by $A \leftrightarrow B$. Then

$$\begin{aligned}\mathbb{P}\{A \leftrightsquigarrow C\} &= \mathbb{P}\{A \leftrightsquigarrow C\,|\,A \leftrightarrow C\}\cdot(1-p) + \mathbb{P}\{A \leftrightsquigarrow C\,|\,A \not\leftrightarrow C\}\cdot p\\ &= 1\cdot(1-p) + \mathbb{P}\{A \leftrightarrow B,\ B \leftrightarrow C\}\cdot p = 1-p+(1-p)^2\cdot p.\end{aligned}$$

9. With the notation of the previous problem, the events $A \leftrightsquigarrow B$ and $B \leftrightsquigarrow C$ are independent. Hence

$$\mathbb{P}\{A \leftrightsquigarrow C\} = \mathbb{P}\{A \leftrightsquigarrow B\}\cdot\mathbb{P}\{B \leftrightsquigarrow C\} = [1-q^2]\cdot(1-q).$$

10. Clearly the probability is zero if $i > 6$.

Without order: If we only consider the sex of the contestants, then each of the $\binom{10}{5}$ possible orders are equally likely. Out of these we have to count how many will give men for the first $i-1$ positions, then a woman on the ith position. In other words we have to count the number of ways 4 women and $5-(i-1)=6-i$ men can be arranged on the $10-i$ positions behind the first woman. The answer is $\binom{10-i}{4}$, and the probability in question is

$$\frac{\binom{10-i}{4}}{\binom{10}{5}}=\frac{(10-i)!\cdot 5!\cdot 5!}{10!\cdot 4!\cdot(6-i)!}=\frac{(10-i)!\cdot 5\cdot 5!}{10!\cdot(6-i)!}.$$

With order: In this case all contestants are considered different. We have to count that, out of the 10! possible orders, in how many cases are there men on the first $i-1$ positions, and a woman on the ith position. For the first $i-1$ positions we can arrange men in $5!/[5-(i-1)]!=5!/(6-i)!$ many ways ($i-1$ permutations of 5 men). Then we have 5 choices for selecting the woman at position i, then $(10-i)!$ ways to arrange the remaining contestants on the $10-i$ positions left. The probability in question is thus

$$\frac{5!\cdot 5\cdot(10-i)!}{(6-i)!\cdot 10!}.$$

The answers for $i=1,\,2,\,3,\,4,\,5$ and 6 are, respectively, $1/2,\ 5/18,\ 5/36,\ 5/84,\ 5/252,\ 1/252$.

11.

$$\mathbb{E}(X)=\sum_{i=1}^{4} i\cdot p(i)=\sum_{i=1}^{4}\frac{i^3}{30}=\frac{1^3}{30}+\frac{2^3}{30}+\frac{3^3}{30}+\frac{4^3}{30}=\frac{10}{3}.$$

12. Let X be the square of the difference of the numbers shown by the two dice. Then X has mass function $p(0)=1/6,\ p(1)=10/36,\ p(4)=8/36,\ p(9)=6/36,\ p(16)=4/36,\ p(25)=2/36$, and expectation

$$\mathbb{E}(X)=0\cdot\frac{1}{6}+1\cdot\frac{10}{36}+4\cdot\frac{8}{36}+9\cdot\frac{6}{36}+16\cdot\frac{4}{36}+25\cdot\frac{2}{36}=\frac{210}{36}=\frac{35}{6}.$$

Similarly, Y is the sum of the two numbers, its mass function is $p(2)=p(12)=1/36,\ p(3)=p(11)=2/36,\ p(4)=p(10)=3/36,\ p(5)=p(9)=4/36,\ p(6)=p(8)=5/36,\ p(7)=6/36$, and its expectation is 7=42/6. Hence the game favors Bob in the long run.

13. The expected return are

$$\frac{1}{40\,000}\cdot\mathrm{HUF}1\,000\,000+\frac{10}{40\,000}\cdot\mathrm{HUF}50\,000+\frac{100}{40\,000}\cdot\mathrm{HUF}5\,000=\mathrm{HUF}50,$$

so tickets should be sold for HUF100.

14. The expected return is

$$\frac{\binom{5}{2}\cdot\binom{85}{3}}{\binom{90}{5}}\cdot 700+\frac{\binom{5}{3}\cdot\binom{85}{2}}{\binom{90}{5}}\cdot 10\,000+\frac{\binom{5}{4}\cdot\binom{85}{1}}{\binom{90}{5}}\cdot 789\,000+\frac{1}{\binom{90}{5}}\cdot 535\,000\,000\simeq \mathrm{HUF}43.^{\underline{66}}.$$

If tickets cost HUF150, then on average we loose HUF106.$^{\underline{34}}$ on each ticket.

15 Ann pays Bob with probability $\frac{1}{2}\cdot\frac{1}{2}=1/4$, and Bob pays Ann with probability $\frac{1}{2}\cdot\frac{1}{2}+\frac{1}{2}\cdot\frac{1}{2}=1/2$. The game is fair if Ann pays twice the amount that Bob pays, e.g. 2 tokens while Bob pays 1 token.

16 In case of an n sided 'die', the mass function of X is $p(i)=1/n$, $i=1,\,2,\,\ldots n$. With the sum formula for the arithmetic sequence,

$$\mathbb{E}(X)=\sum_{i=1}^{n} i\cdot p(i)=\frac{1}{n}\sum_{i=1}^{n} i=\frac{1}{n}\cdot\frac{n(n+1)}{2}=\frac{n+1}{2}.$$

For the standard deviation we need the second moment as well, and we use the sum formula for the square numbers:

$$\mathbb{E}(X^2) = \sum_{i=1}^{n} i^2 \cdot p(i) = \frac{1}{n}\sum_{i=1}^{n} i^2 = \frac{1}{n} \cdot \frac{2n^3 + 3n^2 + n}{6} = \frac{2n^2 + 3n + 1}{6}.$$

The standard deviation is then

$$\mathrm{SD}(X) = \sqrt{\mathbb{E}(X^2) - [\mathbb{E}(X)]^2} = \sqrt{\frac{2n^2 + 3n + 1}{6} - \frac{n^2 + 2n + 1}{4}} = \sqrt{\frac{n^2 - 1}{12}}.$$

For an ordinary die we have $n = 6$, $\mathbb{E}(X) = 7/2$, $\mathrm{SD}(X) = \sqrt{35/12}$.

17.(a) We have a greater chance of choosing a student from a crowded bus, while we pick each bus with equal chance when choosing one of the drivers. Therefore we expect X to be greater than Y.

(b) Using the mass function of X,

$$\mathbb{E}(X) = 40 \cdot \frac{40}{148} + 33 \cdot \frac{33}{148} + 25 \cdot \frac{25}{148} + 50 \cdot \frac{50}{148} \simeq 39.28.$$

As Y takes on any of the given values with equal chance,

$$\mathbb{E}(Y) = 40 \cdot \frac{1}{4} + 33 \cdot \frac{1}{4} + 25 \cdot \frac{1}{4} + 50 \cdot \frac{1}{4} = 37.$$

(c) For the standard deviation we determine the second moments:

$$\mathbb{E}(X^2) = 40^2 \cdot \frac{40}{148} + 33^2 \cdot \frac{33}{148} + 25^2 \cdot \frac{25}{148} + 50^2 \cdot \frac{50}{148} \simeq 1625.4,$$

$$\mathbb{E}(Y^2) = 40^2 \cdot \frac{1}{4} + 33^2 \cdot \frac{1}{4} + 25^2 \cdot \frac{1}{4} + 50^2 \cdot \frac{1}{4} = 1453.5.$$

The standard deviations are then

$$\mathrm{SD}(X) = \sqrt{\mathbb{E}(X^2) - [\mathbb{E}(X)^2]} \simeq \sqrt{1625.4 - 39.28^2} \simeq 9.06,$$

$$\mathrm{SD}(Y) = \sqrt{\mathbb{E}(Y^2) - [\mathbb{E}(Y)^2]} \simeq \sqrt{1453.5 - 37^2} \simeq 9.19.$$

18. The mass function of X is:

$$p(0) = \frac{\binom{4}{0} \cdot \binom{6}{3}}{\binom{10}{3}} = \frac{1}{6}, \quad p(1) = \frac{\binom{4}{1} \cdot \binom{6}{2}}{\binom{10}{3}} = \frac{1}{2}, \quad p(2) = \frac{\binom{4}{2} \cdot \binom{6}{1}}{\binom{10}{3}} = \frac{3}{10}, \quad p(3) = \frac{\binom{4}{3} \cdot \binom{6}{0}}{\binom{10}{3}} = \frac{1}{30}.$$

Then

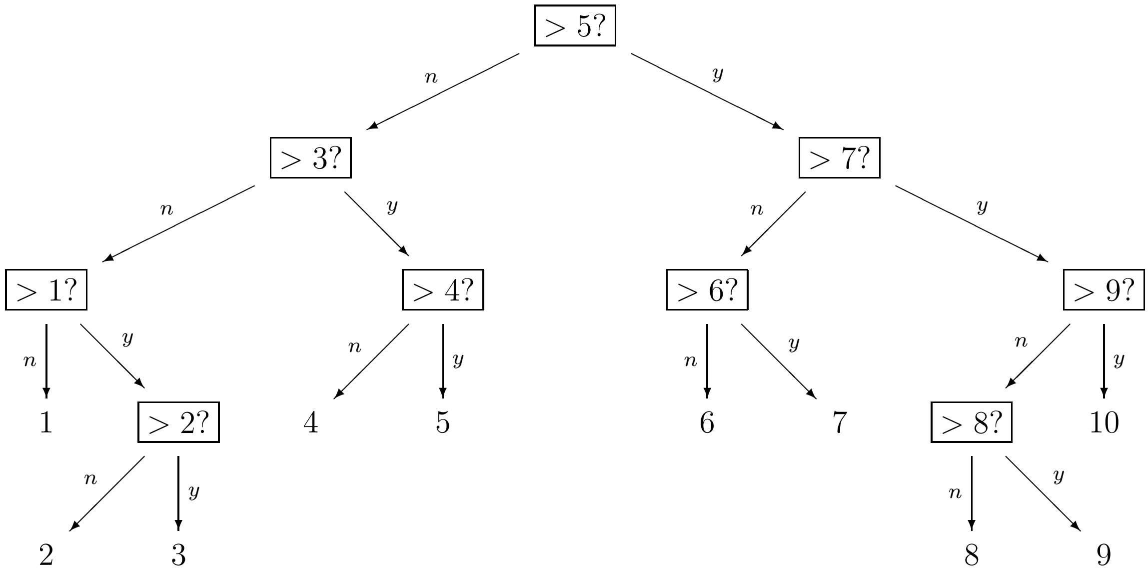
$$\mathbb{E}(X) = 0 \cdot \frac{1}{6} + 1 \cdot \frac{1}{2} + 2 \cdot \frac{3}{10} + 3 \cdot \frac{1}{30} = \frac{6}{5},$$

$$\mathbb{E}(X^2) = 0^2 \cdot \frac{1}{6} + 1^2 \cdot \frac{1}{2} + 2^2 \cdot \frac{3}{10} + 3^2 \cdot \frac{1}{30} = 2,$$

$\mathrm{SD}(X) = \sqrt{\mathbb{E}(X^2) - [\mathbb{E}(X)]^2} = \sqrt{2 - [6/5]^2} = \sqrt{14}/5.$

21.(a) Let X be the random number to be found out. Our method needs the number of questions equal to the value of X. Hence the answer is $\mathbb{E}(X) = 5.5$.

(b) Let our strategy be the following:



Our first question is whether the number is greater than 5. With all the questions we then try to eliminate half of the possibilities left. We need three questions if and only if the random number is $1, 4, 5, 6, 7$, or 10 (that is, with probability 6/10), and four questions if and only if the number is $2, 3, 8$, or 9 (with probability 4/10). The expected number of questions is hence $3 \cdot \frac{6}{10} + 4 \cdot \frac{4}{10} = \frac{17}{5} = 3.4$. For a random number between 1 and 10 the two methods do not show an essential difference in length, but indeed method (b) is much faster for finding out large random numbers.

22.(a) $\mathbb{E}[(2+X)^2] = \mathbb{E}(4)+\mathbb{E}(4X)+\mathbb{E}(X^2) = 4+4\mathbb{E}(X)+\mathbf{Var}(X)+[\mathbb{E}(X)]^2 = 4+4\cdot 1+5+1^2 = 14.$

(b) Additive constants do not matter for the variance, and multiplicative constants are factored out as squares. Therefore $\mathbf{Var}(4 + 3X) = 3^2 \cdot \mathbf{Var}(X) = 9 \cdot 5 = 45$.

23.

$$\mathbb{E}(Y) = \mathbb{E}\Big(\frac{1}{\sigma} \cdot X - \frac{\mu}{\sigma}\Big) = \frac{1}{\sigma} \cdot \mathbb{E}(X) - \frac{\mu}{\sigma} = 0,$$
$$\mathbf{Var}(Y) = \mathbf{Var}\Big(\frac{1}{\sigma} \cdot X - \frac{\mu}{\sigma}\Big) = \frac{1}{\sigma^2}\mathbf{Var}(X) = 1,$$

hence $\mathrm{SD}(Y) = 1$. Y is called the *standardized* of X.

24. The probability that I do not have a common birthday with any of n people is $\left(\frac{364}{365}\right)^n$ (not counting leap years). The probability that at least one of the n people shares birthday with me is $1 - \left(\frac{364}{365}\right)^n$. Hence I need to find n for which

$$\begin{aligned}
1 - \Big(\frac{364}{365}\Big)^n &\geq \frac{1}{2} \\
\Big(\frac{364}{365}\Big)^n &\leq \frac{1}{2} \\
n \cdot \log_2\Big(\frac{364}{365}\Big) &\leq -1 \\
n &\geq \frac{1}{\log_2(365) - \log_2(364)} \simeq 252.7,
\end{aligned}$$

that is at least 253 persons are needed.