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MAT201E DIFFERENTIAL EQUATIONS
WORKSHEET 5
The Laplace Transform, System of first order equations

1. Find the Laplace Transform of the followings;

a. $\mathcal{L}\left[u_3(t)(t^2-2t+1)\right]$

b. $\mathcal{L}\left[u_2(t)(t^2e^{3t})\right]$

c. $\mathcal{L}\left[g(t)\right]$ where $g(t)=\begin{cases}0 & t<3\\ e^{t-3} & t\geq 3\end{cases}$

d. $\mathcal{L}\left[g(t)\right]$ where $g(t)=\begin{cases}0 & t<3\\ 5 & 3\leq t<4\\ t-5 & t\geq 4\end{cases}$

e. $\mathcal{L}\left[g(t)\right]$ where $g(t)=\int_0^t \sin(t-\tau)\cos\tau\, d\tau$

2. Evaluate the following inverse Laplace Transforms;

a. $\mathcal{L}^{-1}\left[\frac{s-2}{s^2-6s+25}\right]$

b. $\mathcal{L}^{-1}\left[e^{-3s}\frac{s}{s^2+8s+18}\right]$

c. $\mathcal{L}^{-1}\left[e^{4s}\frac{1}{s^2-3}\right]$

d. $\mathcal{L}^{-1}\left[e^{s}\frac{1}{(s-3)^2+4}\right]$

e. $\mathcal{L}^{-1}\left[\frac{e^{-2s-2}(s+1)}{s^2+2s+5}\right]$

3. Solve following initial value problems by using Laplace Transform;

a. $y''+2y'+y=e^{-t},\quad y(0)=0,\quad y'(0)=2$

b. $y''-y'+y=te^{t},\quad y(0)=0,\quad y'(0)=0$

c. $3y''+y'+y=g(t),\quad y(0)=0,\quad y'(0)=0$ where $g(t)=\begin{cases}0 & t<4\\ 2 & 4\leq t<10\\ t & t\geq 10\end{cases}$

d. $y''+5y'+4y=g(t),\quad y(0)=0,\quad y'(0)=0$ where $g(t)=\begin{cases}0 & t<1\\ t-1 & 1\leq t<2\\ 3-t & 2\leq t<3\\ 0 & t\geq 3\end{cases}$

e. $y''+3y'+2y=\delta(t-5)+u_{10}(t),\quad y(0)=0,\quad y'(0)=1/2$

f. $y''+4y=\delta(t-\pi)-\delta(t-2\pi),\quad y(0)=0,\quad y'(0)=0$

4. Find the following inverse Laplace transforms by using the convolution integral

a. $\mathcal{L}^{-1}\left[\frac{1}{(s-3)(s^2-6s+10)}\right]$

b. $\mathcal{L}^{-1}\left[\frac{2s}{(s-2)(s^2-4s+5)}\right]$

5. Find the following convolutions;

a. $(4t)*(5t^4)$

b. $(\sin t)*(e^t)$

6. Express the solution of the given initial value problems in terms of a convolution integral

a. $y^{(4)} + 5y'' + 4y = g(t), \quad y(0) = 1, \quad y'(0) = 0, \quad y''(0) = 0, \quad y'''(0) = 0$

b. $y'' + 4y' + 4y = g(t), \quad y(0) = 2, \quad y'(0) = -3$

7. Solve the given problems

a. $\boldsymbol{x}' = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{pmatrix} \boldsymbol{x}, \quad \boldsymbol{x}(0) = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$

b. $t\, \boldsymbol{x}' = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \boldsymbol{x}$

c. $\boldsymbol{x}' = \begin{pmatrix} 1 & -5 \\ 1 & -3 \end{pmatrix} \boldsymbol{x}, \quad \boldsymbol{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

d. $\boldsymbol{x}' = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix} \boldsymbol{x}, \quad \boldsymbol{x}(0) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

e. $\boldsymbol{x}' = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix} \boldsymbol{x} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} e^t$

f. $\boldsymbol{x}' = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix} \boldsymbol{x} + \begin{pmatrix} 0 \\ \cos t \end{pmatrix}, \quad 0 < t < \pi$

8. Find the fundamental matrix for the given system of equations,

a. $\boldsymbol{x}' = \begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix} \boldsymbol{x}$

b. $\boldsymbol{x}' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \boldsymbol{x}$