MAT201E DIFFERENTIAL EQUATIONS

2011-2012 FALL SEMESTER

WORKSHEET IV

Subject: Series Solutions of Second Order Linear Equations, The Laplace Transform

1. Without actually solving the given differential equation, find a lower bound for the radius of convergence of power series about the ordinary point $x_0 = 0$. About the ordinary point $x_0 = 1$.

$$(x^2 - 25)y'' + 2xy' + y = 0$$
$$(x^2 - 2x + 10)y'' + xy' - 4y = 0$$

2. Find the first five nonzero terms in the solution of the given differential equation by using power series about the ordinary point $x_0 = 0$.

$$(x + 2)y'' + xy' - y = 0$$

3. Determine the singular points of the given differential equations. Classify each singular point regular or irregular.

$$(x^2 + x - 6)y'' + (x + 3)y' + (x - 2)y = 0$$
$$(x^2 - 9)^2 y'' + (x + 3)y' + 2y = 0$$

4. Determine the general solution of given differential equation

$$(x - 2)^2 y'' + 5(x - 2)y' + 8y = 0$$

5. Find γ so that the solution of the initial value problem

$$x^2 y'' - 2y = 0, \quad y(1) = 1, \quad y'(1) = \gamma$$

is bounded as $x \to 0$.

6. Show that the given differential equation has a regular singular point at x = 0. Determine the indicial equation, the recurrence relation, and the roots of the indicial equation. Find the series solution ($x > 0$) corresponding to the larger root. If the roots are unequal and do not differ by an integer, find the series solution corresponding to the smaller root also.

$$x^2y'' + xy' + (x - 2)y = 0$$

7. Find the Laplace transform of $f(t) = \cos(at)$, where a is a real constant.

8. Find the Laplace transform of $f(t) = 10\cos(t - \frac{\pi}{6})$.

9. Use Laplace transform to solve the given initial value problems.

$$y'' + 2y' + 5y = 0, \quad y(0) = 2, \quad y'(0) = -1$$
$$y^{IV} - 4y = 0, \quad y(0) = 1, \quad y'(0) = 0, \quad y''(0) = -2, \quad y'''(0) = 0$$

10. Find the inverse Laplace transform of $F(s) = \dfrac{(s-2)e^{-s}}{s^2 - 4s + 3}$.