Subject: Series Solutions of Second Order Linear Equations, The Laplace Transform

1. Without actually solving the given differential equation, find a lower bound for the radius of convergence of power series about the ordinary point \( x_0 = 0 \). About the ordinary point \( x_0 = 1 \).

\[
\begin{align*}
(x^2 - 25)y'' + 2xy' + y &= 0 \\
(x^2 - 2x + 10)y'' + xy' - 4y &= 0
\end{align*}
\]

2. Find the first five nonzero terms in the solution of the given differential equation by using power series about the ordinary point \( x_0 = 0 \).

\[
(x + 2)y'' + xy' - y = 0
\]

3. Determine the singular points of the given differential equations. Classify each singular point regular or irregular.

\[
\begin{align*}
(x^2 + x - 6)y'' + (x + 3)y' + (x - 2)y &= 0 \\
(x^2 - 9)^2y'' + (x + 3)y' + 2y &= 0
\end{align*}
\]

4. Determine the general solution of given differential equation

\[
(x - 2)^2y'' + 5(x - 2)y' + 8y = 0
\]

5. Find \( \gamma \) so that the solution of the initial value problem

\[
x^2y'' - 2y = 0, \quad y(1) = 1, \quad y'(1) = \gamma
\]

is bounded as \( x \to 0 \).
6. Show that the given differential equation has a regular singular point at \( x = 0 \). Determine the indicial equation, the recurrence relation, and the roots of the indicial equation. Find the series solution \((x > 0)\) corresponding to the larger root. If the roots are unequal and do not differ by an integer, find the series solution corresponding to the smaller root also.

\[
x^2y'' + xy' + (x - 2)y = 0
\]

7. Find the Laplace transform of \( f(t) = \cos(at) \), where \( a \) is a real constant.

8. Find the Laplace transform of \( f(t) = 10 \cos(t - \frac{\pi}{6}) \).

9. Use Laplace transform to solve the given initial value problems.

\[
\begin{align*}
y'' + 2y' + 5y &= 0, \quad y(0) = 2, \quad y'(0) = -1 \\
y^{IV} - 4y &= 0, \quad y(0) = 1, \quad y'(0) = 0, \quad y''(0) = -2, \quad y'''(0) = 0
\end{align*}
\]

10. Find the inverse Laplace transform of \( F(s) = \frac{(s - 2)e^{-s}}{s^2 - 4s + 3} \).