MAT201E FALL 2010-2011
DIFFERENTIAL EQUATIONS
WORKSHEET I

1. The velocity, v, of an object of mass m in the fall can be described by the first order differential equation.

$$m\frac{dv}{dt} = mg - kv$$

where k is a positive constant and g is the acceleration due to gravity. For the initial condition, where $v(0) = 0$, show by using the integrating factor method that

$$v = \frac{mg}{k}(1 - e^{-\frac{k}{m}t}) \ .$$

2. Find the general solution of the given differential equation and use it to determine how solutions behave as $t \to \infty$.

   (a) $y' + y = 5\sin(2t)$

   (b) $(1 + t^2)y' + 4ty = (1 + t^2)^{-2}$

3. Solve the Bernoulli differential equation.

$$xy - \frac{dy}{dx} = y^3 e^{-x^2}$$

4. Consider the initial value problem

$$y' - \frac{3}{2}y = 3t + 2e^t, y(0) = y_0.$$

   Find the value of y_0 that separates solutions that grow positively as $t \to \infty$, from those that grow negatively. How does the solution that corresponds to this critical value of y_0 behave as $t \to \infty$.

5. Solve the initial value problem and determine the interval in which the solution is valid.

$$y' = \frac{(1 + 3x^2)}{(3y^2 - 6y)}, \quad y(0) = 1$$

6. Find an integrating factor and solve the given equation.

$$\left(3x + \frac{6}{y}\right) + \left(\frac{x^2}{y} + \frac{3y}{x}\right)\frac{dy}{dx} = 0$$

7. Find the general solution of the following Riccati equation given the particular solution $y = \frac{1}{x}$.

$$x^2(y' + y^2) = a(1 - xy)$$

where a is a real constant.

8. Verify that both $y_1(t) = 1 - t$ and $y_2(t) = \frac{t^2}{4}$ are solutions of the initial value problem.

$$y' = \frac{-t + (t^2 + 4y)^{1/2}}{2}, \quad y(2) = -1$$

Where are these solutions valid?

9. Find the general solution of the following homogeneous differential equations.

   (a) $2xyy' + (x^2 - y^2) = 0$

   (b) $(2\sqrt{xy} - y)dx + xdy = 0$

10. Solve the given initial value problem and determine how the interval in which the solution exists depends on the initial value y_0.

$$y' = \frac{t^2}{y(1 + t^3)}, \quad y(0) = y_0$$