1. The velocity, \( v \), of an object of mass \( m \) in the fall can be described by the first order differential equation.

\[
m \frac{dv}{dt} = mg - kv
\]

where \( k \) is a positive constant and \( g \) is the acceleration due to gravity. For the initial condition, where \( v(0) = 0 \), show by using the integrating factor method that

\[
v = \frac{mg}{k} \left( 1 - e^{-\frac{k}{m}t} \right).
\]

2. Find the general solution of the given differential equation and use it to determine how solutions behave as \( t \to \infty \).

(a) \( y' + y = 5 \sin(2t) \)
(b) \( (1 + t^2)y' + 4ty = (1 + t^2)^{-2} \)

3. Solve the Bernoulli differential equation.

\[
xy - \frac{dy}{dx} = y^3e^{-x^2}
\]

4. Consider the initial value problem

\[
y' - \frac{3}{2}y = 3t + 2e^t, \quad y(0) = y_0.
\]

Find the value of \( y_0 \) that separates solutions that grow positively as \( t \to \infty \), from those that grow negatively. How does the solution that corresponds to this critical value of \( y_0 \) behave as \( t \to \infty \).

5. Solve the initial value problem and determine the interval in which the solution is valid.

\[
y' = \frac{(1 + 3x^2)}{(3y^2 - 6y)}, \quad y(0) = 1
\]
6. Find an integrating factor and solve the given equation.

\[ (3x + \frac{6}{y}) + \left( \frac{x^2}{y} + \frac{3y}{x} \right) \frac{dy}{dx} = 0 \]

7. Find the general solution of the following Riccati equation given the particular solution \( y = \frac{1}{x} \).

\[ x^2(y' + y^2) = a(1 - xy) \]

where \( a \) is a real constant.

8. Verify that both \( y_1(t) = 1 - t \) and \( y_2(t) = \frac{t^2}{4} \) are solutions of the initial value problem.

\[ y' = -t + \frac{(t^2 + 4y)^{1/2}}{2}, \quad y(2) = -1 \]

Where are these solutions valid?

9. Find the general solution of the following homogeneous differential equations.

   (a) \( 2xyy' + (x^2 - y^2) = 0 \)
   
   (b) \( (2\sqrt{xy} - y)dx + xdy = 0 \)

10. Solve the given initial value problem and determine how the interval in which the solution exists depends on the initial value \( y_0 \).

\[ y' = \frac{t^2}{y(1 + t^3)}, \quad y(0) = y_0 \]