MATHEMATICS 201E CRN:..... MIDTERM 19 November 2011

QUESTION 1 The blanks below will be filled by students. (Except the score)

Surname:	Name:	Group Number:	List Number:	**Score**
Signature:	Electronic Post(e-mail) address:	Student Number:		

For the solution of this question please use only the front face and if necessary the back face of this page.

[**10pt**] **a**) State the order of each of the following differential equations and whether it is linear or nonlinear. If it is nonlinear, show explicitly the term which violates linearity.

i) $\mathbf{\frac{d^2y}{dx^2} + y\frac{dy}{dx} + \sin x = 0}$

ii) $\mathbf{\frac{d^3y}{dx^3}\sin^3 x + \frac{dy}{dx}\cos x + y\sin x = 0}$

iii) $\mathbf{\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^3}$

[**15pt**] **b**) Solve the initial value problem $\mathbf{y' + y\cot t = 4\sin t,\ y(-\pi/2) = 0}$ and state the interval in which the solution is valid.

a) i) 2 - Lineer değil - $y\frac{dy}{dx}$

ii) 3 - Lineer

iii) 2 - Lineer değil - $x\left(\frac{dy}{dx}\right)^3$

b) $\mu(t) = e^{\int p(t)dt}$

$\mu(t) = e^{\int \cot t\,dt} = e^{\int \frac{\cos t}{\sin t}dt}$

$\mu(t) = e^{\ln|\sin t|} = \sin t$ //

b) $y'\sin t + y\cos t = 4\sin^2 t$ $(\mu = \sin t)$

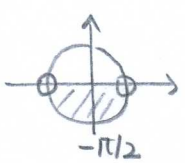
$(y\sin t)' = 4\sin^2 t \Rightarrow y\cdot\sin t = 4\int \sin^2 t\,dt$

$= 2\int (1-\cos 2t)\,dt$

$y\sin t = 2t - \sin 2t + c$

$y = 2t\csc t - 2\cos t + c\csc t$

$y(-\pi/2) = 0 \Rightarrow 0 = +2\frac{\pi}{2} - c \Rightarrow c = \pi \Rightarrow y = (2t+\pi)\csc t - 2\cos t$



$(-\pi, 0)$

QUESTION 2 The blanks below will be filled by students. (Except the score)

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[**12pt**] **a)** Solve the Bernoulli equation $(\mathbf{x^2+1})\mathbf{y'} - \mathbf{xy} = \mathbf{x^3y^3}$.

[**13pt**] **b)** Solve the differential equation $\dfrac{\mathbf{dy}}{\mathbf{dx}} = \dfrac{\mathbf{x-y}}{\mathbf{x-8y+7}}$.

a) $y' - \frac{x}{x^2+1} y = \frac{x^3}{x^2+1} y^3$, $n=3 \Rightarrow v = y^{-2} \Rightarrow v' = -2y^{-3}y'$

$-2y^{-3}y' + \frac{2x}{x^2+1} y^{-2} = -\frac{2x^3}{x^2+1} \Rightarrow v' + \frac{2x}{x^2+1} v = -\frac{2x^3}{x^2+1}$ $(\mu = x^2+1)$

$(x^2+1)v' + 2xv = -2x^3 \Rightarrow [(x^2+1)\cdot v]' = -2x^3 \Rightarrow (x^2+1)\cdot v = -\frac{1}{2}x^4 + c$

$(x^2+1)y^{-2} = -\frac{1}{2}x^4 + c$ $(y \neq 0)$ ($y=0$ is also a solution)

b) $\left.\begin{array}{l} x = X+k \\ y = Y+h \end{array}\right\} \dfrac{dY}{dX} = \dfrac{X-Y+(k-h)}{X-8Y+(k-8h+7)}$, $\left.\begin{array}{l} k-h=0 \\ k-8h=-7 \end{array}\right\} \begin{array}{l} h=1 \\ k=1 \end{array}$

$\frac{dY}{dX} = \frac{1-(Y/X)}{1-8(Y/X)}$, $\frac{Y}{X} = u \Rightarrow u'X + u = \frac{1-u}{1-8u} \Rightarrow u'X = \frac{1-2u+8u^2}{1-8u}$

$\frac{1-8u}{1-2u+8u^2}\,du = \frac{dX}{X} \Rightarrow -\frac{1}{2}\ln|1-2u+8u^2| = \ln(c_1 X)$

$\left|1 - 2\,\frac{y-1}{x-1} + 8\cdot\left(\frac{y-1}{x-1}\right)^2\right| = c_2 (x-1)^{-2}$, $(x \neq 1)$