MATHEMATICS 201E CRN:.......... MIDTERM 19 November 2011

QUESTION 1 The blanks below will be filled by students. (Except the score)

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For the solution of this question please use only the front face and if necessary the back face of this page.

[10pt] **a)** State the order of each of the following differential equations and whether it is linear or nonlinear. If it is nonlinear, show explicitly the term which violates linearity.

i) $\mathbf{\frac{d^2y}{dx^2} + y\frac{dy}{dx} + \sin x = 0}$

ii) $\mathbf{\frac{d^3y}{dx^3}\sin^3 x + \frac{dy}{dx}\cos x + y\sin x = 0}$

iii) $\mathbf{\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^3}$

[15pt] **b)** Solve the initial value problem $\mathbf{y' + y\ \cot t = 4\sin t,\ y(-\pi/2) = 0}$ and state the interval in which the solution is valid.

a) i) 2 - Lineer degil - $y\frac{dy}{dx}$

ii) 3 - Lineer

iii) 2 - Lineer degil - $x\left(\frac{dy}{dx}\right)^3$

b) $\mu(t) = e^{\int p(t)dt}$

$\mu(t) = e^{\int \cot t\, dt} = e^{\int \frac{\cos t}{\sin t}dt}$

$\mu(t) = e^{\ln|\sin t|} = \sin t$ //

b) $\underbrace{y'\sin t + y\cos t} = 4\sin^2 t$ $(\mu = \sin t)$

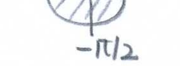
$(y.\sin t)' = 4\sin^2 t \Rightarrow y.\sin t = 4\int \sin^2 t\, dt$

$= 2\int (1-\cos 2t)\,dt$

$y\sin t = 2t - \sin 2t + c$

$y = 2t\csc t - 2\cos t + c\csc t$

$y(-\pi/2) = 0 \Rightarrow 0 = +2\frac{\pi}{2} - c \Rightarrow c = \pi \Rightarrow y = (2t+\pi)\csc t - 2\cos t$



$-\pi/2$

$(-\pi, 0)$

QUESTION 2 The blanks below will be filled by students. (Except the score)

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[**12pt**] **a)** Solve the Bernoulli equation $(\mathbf{x^2+1})\mathbf{y'} - \mathbf{xy} = \mathbf{x^3y^3}$.

[**13pt**] **b)** Solve the differential equation $\dfrac{\mathbf{dy}}{\mathbf{dx}} = \dfrac{\mathbf{x-y}}{\mathbf{x-8y+7}}$.

a) $y' - \dfrac{x}{x^2+1}\, y = \dfrac{x^3}{x^2+1}\, y^3$, $n=3 \Rightarrow v = y^{-2} \Rightarrow v' = -2y^{-3}y'$

$-2y^{-3}y' + \dfrac{2x}{x^2+1}\, y^{-2} = -\dfrac{2x^3}{x^2+1} \Rightarrow v' + \dfrac{2x}{x^2+1}\, v = -\dfrac{2x^3}{x^2+1}$ $(\mu = x^2+1)$

$(x^2+1)v' + 2xv = -2x^3 \Rightarrow [(x^2+1)\cdot v]' = -2x^3 \Rightarrow (x^2+1).v = -\frac{1}{2}x^4 + c$

$(x^2+1)y^{-2} = -\frac{1}{2}x^4 + c$ $(y \neq 0)$ ($y=0$ is also a solution)

b) $\left.\begin{array}{l} x = X + k \\ y = Y + h \end{array}\right\}$ $\dfrac{dY}{dX} = \dfrac{X - Y + (k-h)}{X - 8Y + (k - 8h + 7)}$, $\left.\begin{array}{l} k-h=0 \\ k-8h=-7 \end{array}\right\}$ $\begin{array}{l} h=1 \\ k=1 \end{array}$

$\dfrac{dY}{dX} = \dfrac{1-(Y/X)}{1-8(Y/X)}$, $\dfrac{Y}{X} = u \Rightarrow u'.X + u = \dfrac{1-u}{1-8u} \Rightarrow u'X = \dfrac{1-2u+8u^2}{1-8u}$

$\dfrac{1-8u}{1-2u+8u^2}\, du = \dfrac{dX}{X} \Rightarrow -\frac{1}{2}\ln|1-2u+8u^2| = \ln(c_1 X)$

$\left|1 - 2\,\dfrac{y-1}{x-1} + 8\cdot\left(\dfrac{y-1}{x-1}\right)^2\right| = c_2\,(x-1)^{-2}$, $(x \neq 1)$

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[15pt] a) Show that $\mathbf{y_1(x) = \sin x^2}$ is a solution of $\mathbf{xy'' - y' + 4x^3 y = 0,\ x > 0}$ and find the general solution of this differential equation.

[10pt] b) If $\mathbf{W(f,g)}$ is the Wroskian of $\mathbf{f}$ and $\mathbf{g}$, and if $\mathbf{u = 2f - g,\ v = f + 2g}$, find the Wroskian $\mathbf{W(u,v)}$ of $\mathbf{u}$ and $\mathbf{v}$ in terms of $\mathbf{W(f,g)}$.

a) $y_1 = \sin x^2 \Rightarrow y_1' = 2x\cos x^2 \Rightarrow y_1'' = -4x^2\sin x^2 + 2\cos x^2$

$-x \cdot 4x^2 \sin x^2 + 2x\cos x^2 - 2x\cos x^2 + 4x^3 \sin x^2 = 0$

$v = u y_1 \Rightarrow v' = u'y_1 + u y_1' \Rightarrow v'' = u'' y_1 + 2u'y_1' + u y_1''$

$x(u'' y_1 + 2u'y_1' + \cancel{u y_1''}) - (u'y_1 + \cancel{u y_1'}) + 4x^3 \cancel{u y_1} = 0$

$x(\sin x^2)\, u'' + (4x^2\cos x^2 - \sin x^2)\, u' = 0$

$\dfrac{u''}{u'} = \dfrac{1}{x} - 4x\,\dfrac{\cos x^2}{\sin x^2} \Rightarrow \ln u' = \ln x - 2\ln \sin x^2 + \ln c_1$

$u' = c_1 \dfrac{x}{\sin^2 x^2} \Rightarrow u = -\dfrac{c_1}{2}\cot x^2 + c_2$

$y = -\dfrac{c_1}{2}\cdot \cos x^2 + c_2 \sin x^2$

b) $w(f,g) = \begin{vmatrix} f & g \\ f' & g' \end{vmatrix} = fg' - f'g$

$w(u,v) = \begin{vmatrix} 2f-g & f+2g \\ 2f'-g' & f'+2g' \end{vmatrix} = \cancel{2ff'} + 4fg' - f'g - \cancel{2gg'}$
$- (\cancel{2ff'} - fg' + 4f'g - \cancel{2gg'})$

$= 5fg' - 5f'g = 5\,w(f,g)$

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QUESTION 4 The blanks below will be filled by students. (Except the score)

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[10pt] a) Find the homogeneous solution of $\mathbf{y^{iv} - y = f(t)}$. Determine the particular solution if

i) $\mathbf{f(t) = t^2 + 1}$

ii) $\mathbf{f(t) = 2e^{-t} + (3t+1)\sin t}$

Do not attempt to calculate the coefficients.

[15pt] b) Solve the differential equation $\mathbf{4y'' + y = 2\sec\frac{t}{2}}$, $-\pi < \mathbf{t} < \pi$.

a) $r^4 - 1 = 0 \Rightarrow (r^2-1)(r^2+1) = 0 \Rightarrow r_{1,2} = \pm 1,\ r_{3,4} = \pm i$

$y_h = c_1 e^{t} + c_2 e^{-t} + c_3 \cos t + c_4 \sin t$

i) $y_p = At^2 + Bt + C$ ii) $y_p = Ate^{-t} + t\,[(Bt+C)\cos t + (Dt+E)\sin t]$

b) $4r^2 + 1 = 0 \Rightarrow r_{1,2} = \pm \frac{1}{2} i \Rightarrow y_h = c_1 \cos\frac{t}{2} + c_2 \sin\frac{t}{2}$

$y = u_1 \cos\frac{t}{2} + u_2 \sin\frac{t}{2} \Rightarrow y' = u_1' \cos\frac{t}{2} + u_2' \sin\frac{t}{2} + \frac{1}{2}(-u_1 \sin\frac{t}{2} + u_2 \cos\frac{t}{2})$

$\Rightarrow u_1' \cos\frac{t}{2} + u_2' \sin\frac{t}{2} = 0 \quad (1)$

$\Rightarrow y' = \frac{1}{2}(-u_1 \sin\frac{t}{2} + u_2 \cos\frac{t}{2}) \Rightarrow y'' = \frac{1}{4}(-2u_1' \sin\frac{t}{2} + 2u_2' \cos\frac{t}{2}$
$- u_1 \cos\frac{t}{2} - u_2 \sin\frac{t}{2})$

$4y'' + y = 2\sec\frac{t}{2} \Rightarrow -u_1' \sin\frac{t}{2} + u_2' \cos\frac{t}{2} = \sec\frac{t}{2} \quad (2)$

$(1),(2) \Rightarrow u_2' = 1,\ u_1' = -\tan\frac{t}{2}$

$\Rightarrow u_1 = 2\ln\left|\cos\frac{t}{2}\right| + c_1 = 2\ln(\cos\frac{t}{2}) + c_1 \quad (-\pi/2 < \frac{t}{2} < \pi/2)$

$u_2 = t + c_2$

$y = 2\ln(\cos\frac{t}{2})\cos\frac{t}{2} + t\sin\frac{t}{2} + c_1\cos\frac{t}{2} + c_2\sin\frac{t}{2}$