Math 170 Worksheet 5

1. Find the domains of the following functions. In (b) and (c), sketch the domains.

(a) $f(x, y, z) = \sqrt{4 - x^2 - y^2 - z^2}$ (b) $f(x, y) = \ln(x + y)$ (c) $f(x, y) = \sqrt{xy - 1}$

2. For each of the functions below find the limit as $(x, y) \to (0, 0)$ if it exists.

(a) $\dfrac{\sin x \sin y}{x^2 + y^2}$ (b) $\dfrac{x^3 - y^3}{x^2 + y^2}$ (c) $\dfrac{x^\alpha y^4}{x^2 + y^4}$, $\alpha > 0$ (d) $\dfrac{x^2 + y^4}{x^2 + 2y^4}$ (e) $\dfrac{x - y}{(x^2 + y^2)^\alpha}$, $\alpha < \dfrac{1}{2}$.

Answers:

1. (a) $\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leqslant 4\}$ (this is the closed sphere with center $\vec{0}$ and radius 2) (b) $\{(x, y) \in \mathbb{R}^2 : x + y > 0\}$ (c) $\{(x, y) \in \mathbb{R}^2 : xy > 1\}$. **2.** (a) No limit (approach along $x = 0$ and $y = x$, or use polar coordinates) (b) 0 (c) 0 (d) No limit (approach along $x = 0$ and $y = 0$) (e) 0.