Math 170 Worksheet 5

1. Find the domains of the following functions. In (b) and (c), sketch the domains.
   (a) \( f(x, y, z) = \sqrt{4 - x^2 - y^2 - z^2} \)  
   (b) \( f(x, y) = \ln(x + y) \)  
   (c) \( f(x, y) = \sqrt{xy - 1} \)

2. For each of the functions below find the limit as \((x, y) \to (0, 0)\) if it exists.
   (a) \( \frac{\sin x \sin y}{x^2 + y^2} \)  
   (b) \( \frac{x^3 - y^3}{x^2 + y^2} \)  
   (c) \( \frac{x^\alpha y^4}{x^2 + y^4} \), \( \alpha > 0 \)  
   (d) \( \frac{x^2 + y^4}{x^2 + 2y^4} \)  
   (e) \( \frac{x - y}{(x^2 + y^2)\alpha} \), \( \alpha < \frac{1}{2} \)

Answers:
1. (a) \( \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 4\} \) (this is the closed sphere with center \( \vec{0} \) and radius 2)  
   (b) \( \{(x, y) \in \mathbb{R}^2 : x + y > 0\} \)  
   (c) \( \{(x, y) \in \mathbb{R}^2 : xy > 1\} \).  
2. (a) No limit (approach along \( x = 0 \) and \( y = x \), or use polar coordinates)  
   (b) 0  
   (c) 0  
   (d) No limit (approach along \( x = 0 \) and \( y = 0 \))  
   (e) 0.