Math 170 Worksheet 3

Notation: The line with parametric equation $x = x_0 + ta, y = y_0 + tb, z = z_0 + tc$ is denoted by $\langle x_0 + ta, y_0 + tb, z_0 + tc\rangle$.

1. Let L be a line in $\mathbb{R}^3$ with direction vector $\vec{v}$, Q a point on L and P a point not on L. Show that the distance from P to L is $\dfrac{|\overrightarrow{QP} \times \vec{v}|}{|\vec{v}|}$.
2. Write the parametric equations of the line
   (a) through $(1,2)$ and parallel to $\vec{v} = \langle 2,3\rangle$,
   (b) through $(-2,0,0)$ and parallel to the line segment connecting $P = (0,4,5)$ to $Q = (3,5,-2)$,
   (c) through $(0,4,-4)$ and perpendicular to the plane $x = 2$,
   (d) through $(1,0,1)$ parallel to the planes $x - 2z = 3$ and $y + 3z = x$.
3. Write the equation of the plane
   (a) through $P = (0,0,0), Q = (1,2,3)$ and $R = (-1,1,1)$,
   (b) through $P = (2,0,2)$ and parallel to the plane $3x - 2y + z = 1$,
   (c) through $P = (1,0,0)$ and containing the line $\langle 1 + t, t, 1 - t\rangle, t \in \mathbb{R}$,
   (d) through $P = (8,4,-1)$ and perpendicular to the line $\langle 2t - 1, 3 - t, t + 5\rangle, t \in \mathbb{R}$.
4. Find the point of intersection of the line $L : \langle 2 + t, -t, 3t + 1\rangle$ and $x + 4y - z = 5$.
5. Find the line of intersection of the planes $x + z = 3$ and $-x + y + 2z = 1$.
6. Find the (minimum) distance between the lines $\langle 3t, 2 - t, t\rangle, t \in \mathbb{R}$ and $\langle -t, t + 1, t + 1\rangle, t \in \mathbb{R}$.
7. What is the volume of the parallelpiped determined by the vectors $\vec{u} = \langle 2,0,2\rangle$, $\vec{v} = \langle 1,6,0\rangle$ and $\vec{w} = \langle -3,1,1\rangle$?
8. Find the points of intersection of the line $L : \langle t, 2t, 3 - t\rangle$, $t \in \mathbb{R}$ and the cylinder $x^2 + z^2 = 9$.
9. Do the lines $\langle 3 + t, 2 - 4t, t\rangle$ and $\langle 4 - s, 3 + s, -2 + 3s\rangle$ intersect? If so, where?

Answers:

2. a) $\langle 1 + 2t, 2 + 3t\rangle, t \in \mathbb{R}$ b) $\langle -2 + 3t, t, -7t\rangle, t \in \mathbb{R}$ c) $\langle t, 4, -4\rangle, t \in \mathbb{R}$ d) $\langle 1 + 2t, -t, 1 + t\rangle, t \in \mathbb{R}$.

3. a) $x + 4y - 3z = 0$ b) $3x - 2y + z = 8$ c) $x - y = 1$ d) $2x - y + z = 11$. **4.** $\left(\frac{4}{3}, \frac{2}{3}, -1\right)$.

5. $\langle 3 + t, 4 + 3t, -t\rangle, t \in \mathbb{R}$. **6.** $3/\sqrt{6} = \sqrt{3/2}$. **7.** 50. **8.** (0,0,3) and (3,6,0). **9.** No.