Math 170 Worksheet 3

Notation: The line with parametric equation \( x = x_0 + ta, y = y_0 + tb, z = z_0 + tc \) is denoted by \( (x_0 + ta, y_0 + tb, z_0 + tc) \).

1. Let \( L \) be a line in \( \mathbb{R}^3 \) with direction vector \( \vec{v} \), \( Q \) a point on \( L \) and \( P \) a point not on \( L \). Show that the distance from \( P \) to \( L \) is \( \frac{|\overrightarrow{QP} \times \vec{v}|}{|\vec{v}|} \).

2. Write the parametric equations of the line
   (a) through \( (1, 2) \) and parallel to \( \vec{v} = (2, 3) \),
   (b) through \( (-2, 0, 0) \) and parallel to the line segment connecting \( P = (0, 4, 5) \) to \( Q = (3, 5, -2) \),
   (c) through \( (0, 4, -4) \) and perpendicular to the plane \( x = 2 \),
   (d) through \( (1, 0, 1) \) parallel to the planes \( x - 2z = 3 \) and \( y + 3z = x \).

3. Write the equation of the plane
   (a) through \( P = (0, 0, 0), Q = (1, 2, 3) \) and \( R = (-1, 1, 1) \),
   (b) through \( P = (2, 0, 2) \) and parallel to the plane \( 3x - 2y + z = 1 \),
   (c) through \( P = (1, 0, 0) \) and containing the line \( (1 + t, t, 1 - t), t \in \mathbb{R} \),
   (d) through \( P = (8, 4, -1) \) and perpendicular to the line \( (2t - 1, 3 - t, t + 5), t \in \mathbb{R} \).

4. Find the point of intersection of the line \( L : (2 + t, -t, 3t + 1) \) and \( x + 4y - z = 5 \).

5. Find the line of intersection of the planes \( x + z = 3 \) and \( -x + y + 2z = 1 \).

6. Find the (minimum) distance between the lines \( (3t, 2 - t, t), t \in \mathbb{R} \) and \( (-t, t + 1, t + 1), t \in \mathbb{R} \).

7. What is the volume of the parallelepiped determined by the vectors \( \vec{u} = (2, 0, 2), \vec{v} = (1, 6, 0) \) and \( \vec{w} = (-3, 1, 1) \)?

8. Find the points of intersection of the line \( L : (t, 2t, 3 - t), t \in \mathbb{R} \) and the cylinder \( x^2 + z^2 = 9 \).

9. Do the lines \( (3 + t, 2 - 4t, t) \) and \( (4 - s, 3 + s, -2 + 3s) \) intersect? If so, where?

Answers:

2. a) \( (1 + 2t, 2 + 3t), t \in \mathbb{R} \) b) \( (-2 + 3t, t, -7t), t \in \mathbb{R} \) c) \( (t, 4, -4), t \in \mathbb{R} \) d) \( (1 + 2t, -t, 1 + t), t \in \mathbb{R} \).

3. a) \( x + 4y - 3z = 0 \) b) \( 3x - 2y + z = 8 \) c) \( x - y = 1 \) d) \( 2x - y + z = 11 \) e) \( (\frac{5}{3}, 2, -1) \).

5. \( (3 + t, 4 + 3t, -t), t \in \mathbb{R} \). 6. \( 3/\sqrt{6} = \sqrt{3}/2 \). 7. 50. 8. \((0,0,3)\) and \((3,6,0)\). 9. No.