

Math 170 - Worksheet 1

1. Sketch the following regions in \mathbb{R}^2 :

- (a) $y \geq 0, x^2 + y^2 \leq 2$.
 (b) $y \leq x, y \geq x^2 - 1$.
 (c) $xy \geq 1$ (be careful!).
 (d) $-1 \leq x \leq 1, x - 1 \leq y \leq \sqrt{1 - x^2}$.

2. Describe the given regions in \mathbb{R}^2 using inequalities as in Problem 1:

- (a) The region outside the circle $x^2 + y^2 = 1$ and inside the circle with center $(0,0)$ and radius 5.
 (b) The region bounded by the coordinate axes and the parabola $y = -x^2 + 4x + 5$.
 (c) The trapezoid with vertices $(0,0), (2,0), (-2,1)$ and $(6,1)$.
 (d) The region inside the circle with center $(1,1)$, radius 1 and above the line $y = 2x$.
 (e) The region bounded by the parabola $x = y^2$ and the line $x + y = 2$.

3. Describe the given regions in \mathbb{R}^2 in an alternative form:

- (a) $0 \leq x \leq 1, x^3 \leq y \leq x^2$.
 (b) $-1 \leq x \leq 1, \sqrt{3} \leq y \leq \sqrt{4 - x^2}$.
 (c) $1 \leq x \leq e, \ln x \leq y \leq 1$.
 (d) The region bounded by $xy = 1, x + y = 3$.

4. Sketch the following surfaces or regions in \mathbb{R}^3 :

- (a) $z = \sqrt{x^2 + y^2}$.
 (b) $z \geq x^2 + y^2, z \leq 4$.
 (c) $x^2 + y^2 + z^2 \leq 1, y + z \geq 0$.

5. Describe the regions in \mathbb{R}^3 given below using inequalities:

- (a) The region in $z \geq 0$ bounded by $x^2 + y^2 = 1$ and $y - z = 0$.
 (b) The region bounded by $x = 0, y = 0, x + y = 1$ and $x^2 + y^2 + z^2 = 1$.
 (c) The region bounded by $x = 0, y = 0, z = 0, x + z = 1$ and $y = 1 - x^2$.

Answers:

- 2.** (a) $1 \leq x^2 + y^2 \leq 25$ (b) $0 \leq x \leq 5, 0 \leq y \leq -x^2 + 4x + 5$ (c) $0 \leq y \leq 1, -2y \leq x \leq 4y + 2$
 (d) $\frac{2}{5} \leq y \leq 2, 1 - \sqrt{1 - (y - 1)^2} \leq x \leq \frac{y}{2}$ (one can simplify $\sqrt{1 - (y - 1)^2} = \sqrt{2y - y^2}$)
 (e) $-2 \leq y \leq 1, y^2 \leq x \leq 2 - y$. **3.** (a) $0 \leq y \leq 1, \sqrt{y} \leq x \leq \sqrt[3]{y}$ (b) $\sqrt{3} \leq y \leq 2, -\sqrt{4 - y^2} \leq x \leq \sqrt{4 - y^2}$ (c) $0 \leq y \leq 1, 1 \leq x \leq e^y$ (d) $\frac{3 - \sqrt{5}}{2} \leq x \leq \frac{3 + \sqrt{5}}{2}, \frac{1}{x} \leq y \leq 3 - x$, or $\frac{3 - \sqrt{5}}{2} \leq y \leq \frac{3 + \sqrt{5}}{2}, \frac{1}{y} \leq x \leq 3 - y$.
5. (a) $0 \leq y \leq 1, -\sqrt{1 - y^2} \leq x \leq \sqrt{1 - y^2}, 0 \leq z \leq y$, or $-1 \leq x \leq 1, 0 \leq y \leq \sqrt{1 - x^2}, 0 \leq z \leq y$.
 (b) $0 \leq x \leq 1, 0 \leq y \leq 1 - x, -\sqrt{1 - x^2 - y^2} \leq z \leq \sqrt{1 - x^2 - y^2}$, or $0 \leq y \leq 1, 0 \leq x \leq 1 - y, -\sqrt{1 - x^2 - y^2} \leq z \leq \sqrt{1 - x^2 - y^2}$.
 (c) $0 \leq x \leq 1, 0 \leq y \leq 1 - x^2, 0 \leq z \leq 1 - x$, or $0 \leq y \leq 1, 0 \leq x \leq \sqrt{1 - y}, 0 \leq z \leq 1 - x$, or $0 \leq z \leq 1, 0 \leq x \leq 1 - z, 0 \leq y \leq 1 - x^2$.