MATHEMATICS 101E MIDTERM NOVEMBER 19, 2011

QUESTION 1 The blanks below will be filled by students. (Except the score)

Surname:	Name:	Group Number:	List Number:	**Score**
Signature:	Electronic Post(e-mail) address:	Student Number:		

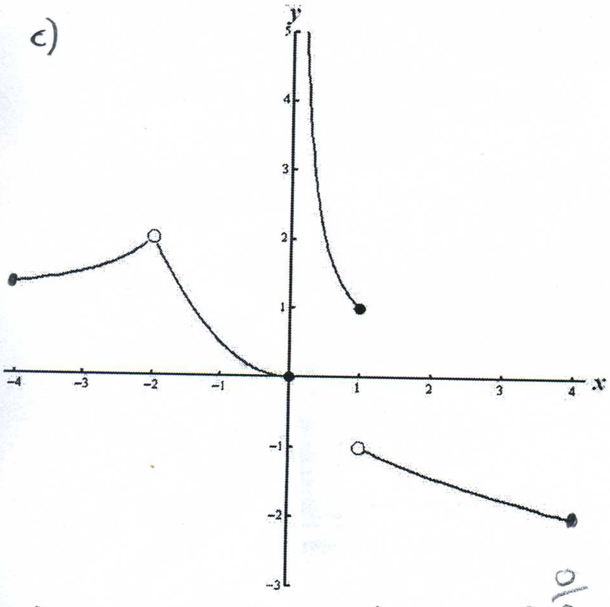
For the solution of this question please use only the front face and if necessary the back face of this page.

[5 pt] a) Graph the function of $\mathbf{f(x) = e^{x+1} - 2}$ by using horizontal and vertical shifts.

[10 pt] b) $\lim\limits_{x\to 4} \dfrac{(\sqrt{x^2-7} - x + 1)\sin(x-4)}{(x^2-3x-4)^2}$ =? (Do not use the L'Hopital's Rule)

[10 pt] c) Let $\mathbf{f(x)}$ be the function illustrated in the figure below. For what values of $\mathbf{x}$ is $\mathbf{f(x)}$ discontinuous on $[-4, 4]$? Classify the types of discontinuities. Give reasons for your answer.

c)



f is discontinuous at $x=-2, 0$ and 1.

at $x=-2$: $\lim\limits_{x\to -2^-} f(x) = 2$, $\lim\limits_{x\to -2^+} f(x) = 2$

f has a removable discontinuity at $x=-2$

at $x=0$ $\lim\limits_{x\to 0^-} f(x) = 0$, but $\lim\limits_{x\to 0^+} f(x) = \infty$, therefore,

f has an infinite discontinuity at $x=0$

at $x=1$, since $\lim\limits_{x\to 1^-} f(x) = 1 \neq \lim\limits_{x\to 1^+} f(x) = -1$,

f has a jump discontinuity at $x=1$.

b) $\lim\limits_{x\to 4} \dfrac{(\sqrt{x^2-7} - (x-1))\sin(x-4)}{(x-4)^2(x+1)^2} \overset{0/0}{=} \lim\limits_{x\to 4} \dfrac{\sin(x-4)}{(x-4)} \cdot \lim\limits_{x\to 4} \dfrac{[\sqrt{x^2-7}-(x-1)][\sqrt{x^2-7}+(x-1)]}{(x-4)(x+1)^2[\sqrt{x^2-7}+(x-1)]}$

$= \underbrace{\lim\limits_{x\to 4} \dfrac{\sin(x-4)}{(x-4)}}_{1} \cdot \lim\limits_{x\to 4} \dfrac{x^2-7-(x^2-2x+1)}{(x-4)(x+1)^2[\sqrt{x^2-7}+(x-1)]} = \lim\limits_{x\to 4} \dfrac{2\cancel{(x-4)}}{\cancel{(x-4)}(x+1)^2[\sqrt{x^2-7}+(x-1)]}$

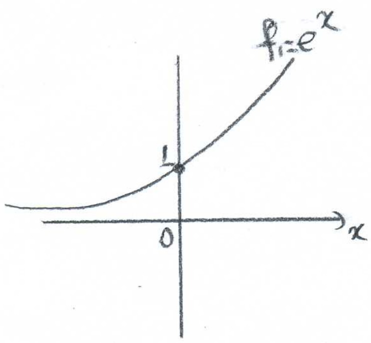
3 3

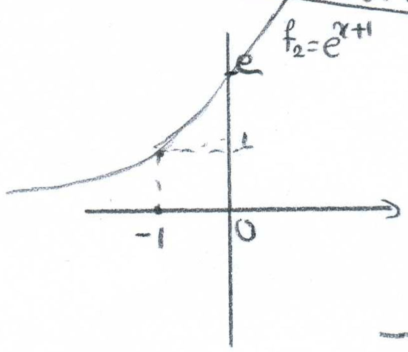
$= \dfrac{2}{25.6} = \dfrac{1}{75}$ //

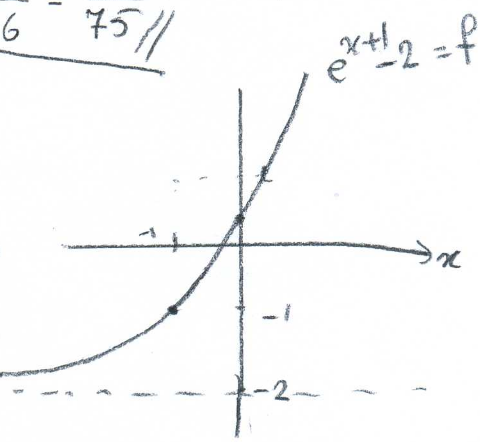
a) $f_1 = e^x$

$f_2 = e^{x+1}$

$f = e^{x+1} - 2$







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QUESTION 2 The blanks below will be filled by students. (Except the score)

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[10 pt] a) Show that the function $\mathbf{f(x)} = \dfrac{\mathbf{1}}{\mathbf{x}}$ satisfies the Mean Value Theorem on the interval $[\mathbf{a}, \mathbf{b}]$ such that $\mathbf{a}, \mathbf{b} > \mathbf{0}$ and find the value of $\mathbf{c}$ in the conclusion of the theorem.

[15 pt] b) Find the equation of the normal line of the curve $\sin(\mathbf{2xy}) + \mathbf{y^2} - \mathbf{2xy} - \tan(\pi\mathbf{x}) = \mathbf{1}$ at the point $\mathbf{P(0,1)}$.

a) The Mean Value Theorem:
If $y=f(x)$ i) is continuous on $[a,b]$
ii) is differentiable in (a,b)
then, there is at least one point c in (a,b) at which $f'(c)=\dfrac{f(b)-f(a)}{b-a}$

$f(x)=\dfrac{1}{x}$, $a,b>0$

i) Is f continuous on $[a,b]$
f is undefined at $x=0 \notin [a,b] \Rightarrow f$ is con.

ii) Is f differentiable in (a,b)?
Yes, $f'(x)=\dfrac{-1}{x^2}$, $x\neq 0$

There is at least one point c in (a,b);

$$f'(c)=\frac{f(b)-f(a)}{b-a} \Rightarrow -\frac{1}{c^2} = \frac{\frac{1}{b}-\frac{1}{a}}{b-a} \Rightarrow -\frac{1}{c^2} = \frac{\overset{(-1)}{\cancel{a-b}}}{ab\,\cancel{(b-a)}} \Rightarrow c^2 = ab$$

$c=\sqrt{ab}$ $\quad$ $\cancel{c=-\sqrt{ab}}$

$c\in(a,b)$, $0<a<b$.

b) $\sin(2xy)+y^2-2xy-\tan(\pi x)=1$, $y=y(x)$

$\Rightarrow (2xy)'.\cos(2xy)+2yy'-2.(xy)'-\pi\sec^2(\pi x)=0$

$\Rightarrow 2[y+xy']\cos(2xy)+2yy'-2[y+xy']-\pi\sec^2(\pi x)=0$

$\Rightarrow y'[2x\cos(2xy)+2y-2x]=\pi\sec^2(\pi x)-2y\cos(2xy)+2y$

$$y'=\frac{\pi\sec^2(\pi x)-2y\cos(2xy)+2y}{2x\cos(2xy)+2y-2x} \Rightarrow m_{tangent}=y'\Big|_{P(0,1)}=\frac{\pi\sec^2 0-2.\cos 0+2}{0+2-0}=\frac{\pi}{2}$$

$$m_{normal}=-\frac{1}{m_{tangent}}=-\frac{1}{\pi/2}=-\frac{2}{\pi}$$

The normal line equation: $y-y_0=m_{normal}(x-x_0)$

$$y-1=-\frac{2}{\pi}(x-0) \Rightarrow y=\frac{-2}{\pi}x+1$$