

Exercises with Limits
(Sections 2.3 and 2.6)

1. Evaluate $\lim_{x \rightarrow 4} \frac{x^2 - x - 12}{x - 4}$
2. Evaluate $\lim_{x \rightarrow 2} \frac{x^3 - 5x^2 + 2x - 4}{x^2 - 3x + 3}$
3. Find $\lim_{x \rightarrow \infty} \frac{2x + 5}{x^2 - 7x + 3}$
4. Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{x+3} - \sqrt{3}}{x}$
5. Evaluate $\lim_{x \rightarrow -\infty} (2x^3 - 12x^2 + x - 7)$
6. Evaluate $\lim_{x \rightarrow 2} \left(\frac{1}{x-2} - \frac{4}{x^2-4} \right)$
7. Evaluate $\lim_{x \rightarrow 3} \frac{1}{(x-3)^2}$
8. Find $\lim_{x \rightarrow \infty} \frac{4x-1}{\sqrt{x^2+2}}$
9. Find $\lim_{x \rightarrow -\infty} \frac{4x-1}{\sqrt{x^2+2}}$
10. Find $\lim_{x \rightarrow 2^+} f(x)$ and $\lim_{x \rightarrow 2^-} f(x)$ if:

$$f(x) = \begin{cases} 7x - 2, & \text{if } x \geq 2 \\ 3x + 5, & \text{if } x < 2 \end{cases}$$
11. Find $\lim_{x \rightarrow 2} \frac{\sqrt{x+2} - \sqrt{2x}}{x^2 - 2x}$
12. Find $\lim_{x \rightarrow 2^+} f(x)$ and $\lim_{x \rightarrow 2^-} f(x)$ if $f(x) = \frac{|x-2|}{x-2}$
13. Find $\lim_{x \rightarrow \infty} \tan^{-1}(2x+1)$
14. Find $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$, if $f(x) = x^2 - 4x$.
15. Find all vertical and horizontal asymptotes for $\sqrt{x+1} - \sqrt{x}$
16. Find all vertical and horizontal asymptotes for $\frac{x^2-5x+6}{x-3}$
17. Find all vertical and horizontal asymptotes for $\frac{2x+3}{\sqrt{x^2-2x-3}}$
18. Evaluate $\lim_{x \rightarrow 2} \frac{\sqrt{x^2+5} - 3}{x^2 - 2x}$
19. Evaluate $\lim_{x \rightarrow \infty} (\sqrt{x^2+2x} - x)$
20. Find $\lim_{x \rightarrow \pm\infty} \frac{7x^3 + 2x^2}{4x^3 - x}$
21. Find $\lim_{x \rightarrow \infty} \frac{2x+1}{\sqrt[3]{x^3-2}}$
22. Find $\lim_{x \rightarrow -\infty} \frac{3x+2}{\sqrt{x^2-1}}$
23. Evaluate: $\lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{|x|} \right)$
24. Let

$$h(x) = \begin{cases} x, & \text{if } x < 0 \\ x^2, & \text{if } 0 < x \leq 2 \\ 8-x, & \text{if } x > 2 \end{cases}$$
 Evaluate the following, if they exist.
 - (a) $\lim_{x \rightarrow 0^+} h(x)$
 - (b) $\lim_{x \rightarrow 0^-} h(x)$
 - (c) $\lim_{x \rightarrow 1} h(x)$
 - (d) $\lim_{x \rightarrow 2^+} h(x)$
 - (e) $\lim_{x \rightarrow 2^-} h(x)$
 - (f) $\lim_{x \rightarrow 2} h(x)$
25. Evaluate $\lim_{x \rightarrow 2} \frac{\sqrt{6-x} - 2}{\sqrt{3-x} - 1}$ (This one is a little more difficult!)
26. Show that $\lim_{x \rightarrow 0} \sqrt{x} e^{\sin(\pi/x)} = 0$
27. Evaluate $\lim_{x \rightarrow -\infty} \frac{r^4 - r^2 + 1}{r^5 + r^3 - r}$
28. Evaluate $\lim_{x \rightarrow \infty} \frac{1 - \sqrt{x}}{1 + \sqrt{x}}$
29. Evaluate $\lim_{x \rightarrow -\infty} \frac{6t^2 + 5t}{(1-t)(2t-3)}$
30. Evaluate $\lim_{x \rightarrow \infty} \frac{7t^3 + 4t}{2t^3 - t^2 + 3}$